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INFLUENCE OF PHASE TRANSITIONS ON SOUND PROPAGATION IN FOGS:
COMPARISON OF THEORY WITH EXPERIMENT

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Several theoretical and experimental papers [1-10] have been devoted to the propagation of acoustic disturbances in two-component mixtures of a gas with vapor and liquid droplets. Here we give a brief survey of the latest theoretical publications. We discuss the existing experimental data. We also compare the theory developed in [9] with the experimental data of other authors.

1. The work of Cole and others [2, 4, 5] can evidently be cited among the earliest theoretical studies of the propagation of low-intensity waves in two-component two-phase mixtures of an inert gas with a vapor and liquid droplets in the presence of mass transfer by diffusion. These authors investigated the case of small mass contents of the condensed phase, $m \ll 1$. It was established [2, 5] that the first maximum of the attenuation per wavelength σ in aerosols with phase transitions occurs at $\omega\tau_v \sim m$ (ω is the angular frequency, and τ_v is the Stokes relaxation time of the phase velocities; see Sec. 3 below), i.e., at $\omega\tau_v \ll 1$. The attenuation coefficient in the vicinity of frequencies $\omega\tau_v \sim m$ is much greater than the corresponding values of σ for aerosols without phase transitions. A previous comparison [3] of theory with experiment indicated only qualitative agreement between them.

Marble and Candel [6] investigated the feasibility of using a cloud of fine droplets to attenuate noise with the injection of liquid into the air intake of a turbojet engine. The magnitude of such attenuation is proportional to the vapor concentration k_v in the gaseous phase, but this rule does not hold for large values of k_v . The shortcoming of [2, 4-6] lies in the failure to take into account the difference between the gas constants of the vapor and gas components in the equation of state for the host phase. It was actually assumed, therefore, that the gaseous phase is a calorically ideal gas when mass transfer is present in the disperse system. Allowance for the indicated difference in [7] improved the agreement between theory and the experimental data. However, this agreement still fell short.

All of the cited investigations of sound propagation in vapor-gas-droplet systems were carried out within the framework of a quasiequilibrium phase transition scheme, where it is assumed that the temperature of the droplet surface during mass transfer is equal to the saturation temperature at the given partial pressure of the vapor. The transient effects of phase interaction are significant for high-frequency disturbances in the suspension; when they are taken into account, in general, the effects of nonequilibrium of the phase interface in phase transition are also taken into account. The influence of the sum total of transient and nonequilibrium effects of interphase mass, momentum, and energy transfer on the propagation of acoustic disturbances in mixtures of a gas with vapor and liquid droplets was first investigated by Gubaidullin and Ivandaev [9, 10]. They analyzed the individual contributions of nonequilibrium interphase heat and mass transfer and friction of the phases to wave dispersion and dissipation.

From the experimental point of view, the propagation of weak disturbances in gas suspensions has not been adequately studied to date. The majority of experimental studies have been concerned with sound propagation in suspensions without phase transitions. Accordingly, although the most important data are those pertaining to the influence of phase transition on the dispersion relations, such data are very limited.

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The first quantitative data on sound attenuation in a polydisperse atmospheric fog formed by the atomization of water in air were reported by Knudsen et al. [1]. The experiments were carried out for droplets with diameters up to 50 μm , and the frequency of the disturbances was 500 Hz. However, phase transition was not observed to have any appreciable influence on sound propagation. This is because the characteristic time constant of interphase heat and mass transfer in the sonic irradiation of vapor-gas-droplet systems is τ_m ($\tau_m \sim \tau_v/m$; see Sec. 3 below). In the case of ordinary atmospheric fogs with a mass content of the condensed phase $m \sim 10^{-3}$ to 10^{-2} the time constant τ_m is much smaller than the characteristic times in the absence of phase transitions. Consequently, the frequencies at which the influence of interphase heat and mass transfer on the attenuation and dispersion of sound is a maximum ($\omega\tau_m \sim 1$) can occur in the range of very low frequencies well below 500 Hz. The dimensionless frequency $\omega\tau_m$ in the reported experiments [1] was much greater than unity ($\omega\tau_m \gg 1$). For this reason, the influence of phase transitions on the attenuation per wavelength σ is inconsequential, and σ differs very little from the corresponding value without mass transfer between phases.

Sound attenuation measurements in a mixture of air with water vapor and droplets in the range of dimensionless frequencies $\omega\tau_m \sim 1$ are reported in [3]. A Wilson cloud chamber was used in the experiments to generate a monodisperse fog with a mass content of droplets $m \sim 10^{-2}$. The diameter of the droplets in different tests varies from 2 μm to 10 μm , and the frequency of the disturbances was 80 Hz. The maximum attenuation per wavelength occurred at dimensionless frequencies $\omega\tau_m \sim 1$ ($\omega\tau_v \sim m$). Phase transition was observed to have strong effects on σ : the maximum value of σ was 100 times the value of σ without phase transition. The sound velocity was not measured in the experiments of [1, 3].

2. We use the model of a two-velocity three-temperature continuum [11] to study the phenomenon in the case of an acoustic homogeneity of the investigated monodisperse mixture. We write the linearized equations of planar one-dimensional motion in the presence of phase transitions. In a coordinate system wherein the undisturbed mixture is at rest, the equations of conservation of mass, momentum, and energy of the phases have the form [9]

$$\begin{aligned} \frac{\partial \rho'_1}{\partial t} + \rho_{10} \frac{\partial v'_1}{\partial x} &= -n_0 j_{V\Sigma}, & \frac{\partial \rho'_V}{\partial t} + \rho_{V0} \frac{\partial v'_V}{\partial x} &= -n_0 j_{V\Sigma}, & \frac{\partial \rho'_2}{\partial t} + \rho_{20} \frac{\partial v'_2}{\partial x} &= \\ &= n_0 j_{\Sigma}, & \rho_{10} \frac{\partial v'_1}{\partial t} + \frac{\partial p'_1}{\partial x} + n_0 f &= 0, & \rho_{20} \frac{\partial v'_2}{\partial t} = n_0 f, & \rho_{V0} \frac{\partial i'_V}{\partial t} + \rho_{G0} \frac{\partial i'_G}{\partial t} = \\ &= \alpha_{10} \frac{\partial p'_1}{\partial t} - n_0 q_{1\Sigma}, & \rho_{20} \frac{\partial u'_2}{\partial t} &= -n_0 q_{2\Sigma}, & q_{1\Sigma} + q_{2\Sigma} &= -j_{\Sigma} l_0, & j_{V\Sigma} = j_{\Sigma}, & \rho_{10} = \\ &= \alpha_{10} \rho_{10}^0, & \rho_{20} = \alpha_{20} \rho_{20}^0, & \alpha_{10} + \alpha_{20} = 1, & \alpha_{20} = \frac{4}{3} \pi \alpha_0^3 n_0, & \rho_{10} = \rho_{V0} + \rho_{G0}, & p_{10} = p_{V0} + p_{G0}. \end{aligned} \quad (2.1)$$

Here ρ , ρ^0 , v , and p are the normalized and true densities, the velocity, and the pressure; α is the content by volume; n is the number of particles of radius a in unit volume; f is the force exerted on an individual droplet by the host phase; $j_{V\Sigma}$ is the diffusion flux of vapor toward the droplet surface Σ ; and j_{Σ} is the rate of condensation on the droplet surface. The symbols i , u , and l denote the specific enthalpy, internal energy, and heat of vaporization; $q_{j\Sigma}$ is the heat-transfer rate of the j -th phase from the surface of the droplet ($j = 1, 2$). The subscripts 1 and 2 refer to the parameters of the gaseous and suspended phases; V and G refer to the parameters of the vapor and gas components of the host phase. The prime is used everywhere to signify perturbations of the parameters, and the subscript 0 corresponds to the initial unperturbed state.

We assume that the components of the gaseous phase are calorically ideal gases. The equations of state of the vapor, the gas mixture as a whole, and the incompressible disperse phase can then be written in the linearized form

$$\begin{aligned} p'_V/p_{V0} &= \rho'_V/\rho_{V0} + T'_1/T_0, & i'_V &= c_{pV} T'_1, \\ p'_1/p_{10} &= \rho'_1/\rho_{10} + T'_1/T_0 + R'_1/R_{10}, & \rho'_2 &= 0, & u'_2 &= c_2 T'_2, \end{aligned} \quad (2.2)$$

where R and T are the gas constant and the temperature, and c_{pV} and c_2 are the specific heats of the vapor (at constant pressure) and the disperse phase. We will drop the subscript 0 from now on wherever the meaning is clear.

The system of equations (2.1), (2.2) is closed and can be used to analyze the propagation of acoustic disturbances in mixtures of an inert gas with a vapor and liquid droplets,

provided the intensities of phase interaction f , j_Σ , $j_{V\Sigma}$, and $q_{j\Sigma}$ ($j = 1, 2$) are specified. The force interaction of the phases was determined by the standard technique with allowance for the fact that the main forces acting on an individual particle of the disperse phase are the Stokes and Basset forces [12]. The external and internal heat fluxes $q_{1\Sigma}$ and $q_{2\Sigma}$ of an inclusion toward its surface and the rate of interphase mass transfer $j_{V\Sigma}$ were given by the relations [11]

$$q_{j\Sigma} = 2\pi a \lambda_j \text{Nu}_j (T_j - T_\Sigma), \quad \text{Nu}_j = 2a\beta_j^T / \lambda_j, \quad j = 1, 2,$$

$$(1 - k_V)j_{V\Sigma} = 2\pi a \rho_1^0 D_1 \text{Sh}_1 (k_V - k_{V\Sigma}), \quad \text{Sh}_1 = 2a\beta_1^D / D_1.$$

Here Nu_j and β_j^T are the dimensionless (Nusselt number) and dimensioned heat-transfer coefficients between the j -th phase and the phase interface; Sh_1 and β_1^D are the dimensionless (Sherwood number) and dimensioned mass-transfer coefficients between the host phase and the surface (Σ) layer of the droplet; λ and D are the thermal conductivity and the binary diffusion coefficient; and $k_V = \rho_V / \rho_1$ is the vapor concentration in the gaseous phase.

We note that the dependence of the heat fluxes $q_{j\Sigma}$ ($j = 1, 2$) and the mass-transfer rate $j_{V\Sigma}$ on the wave frequency ω must be taken into account in the investigation of high-frequency irradiation of a gas mixture with phase transitions. This can be done in the three-temperature model of interphase heat transfer for the adopted phase transition scheme by taking into account the frequency dependence of the corresponding dimensionless heat- and mass-transfer coefficients Nu_j ($j = 1, 2$) and Sh_1 [8, 12].

If nonequilibrium phase transition takes place at the phase interface, the vapor pressure $p_{V\Sigma}$ at the boundary differs from the saturated vapor pressure $p_{VS}(T_\Sigma)$ [or, equivalently, the temperature T_Σ of the droplet surface differs from the saturation temperature $T_S(p_{V\Sigma})$]. The rate of nonequilibrium condensation at the phase interface was specified by the Hertz-Knudsen-Langmuir equation [11]

$$\frac{n_{j\Sigma}}{\rho_1} = \frac{1}{\tau_\beta} \frac{p'_{V\Sigma} - p'_{VS}}{p_1}, \quad \tau_\beta = \frac{\alpha_1}{3\alpha_2} \sqrt{\frac{2\pi}{\gamma_V} \frac{\gamma_1 C_V a}{\beta C_1^2}}, \quad p'_{VS} = \left(\frac{dp_{VS}}{dT} \right) T'_\Sigma,$$

where τ_β is a characteristic equalization time of the partial vapor pressure at the phase interface (it depends on the accommodation coefficient β), C is the sound velocity, and γ is the adiabatic index. The subscript S refers to the values of parameters on the phase equilibrium line.

3. According to the dispersion relation [10], for small mass contents of droplets ($m = \rho_2 / \rho_1 \ll 1$) the attenuation per wavelength $\sigma = 2\pi K_{**} C_p / \omega$, the linear attenuation coefficient K_{**} (per unit length), and the phase velocity C_p of the waves are given by the expressions

$$\sigma = \pi (C_p / C_1)^2 I(\omega), \quad K_{**} = (\omega / 2C_1) (C_p / C_1) I(\omega),$$

$$\left(\frac{C_p}{C_1} \right)^2 = \frac{1}{1 + \text{Re} \{ D^0(\omega) + mV^0(\omega) \}}, \quad I(\omega) = \text{Im} \{ D^0(\omega) + mV^0(\omega) \}$$

[$V^0(\omega)$ and $D^0(\omega)$ are complex functions describing dispersion and dissipation effects due to interphase friction and interphase heat and mass transfer, respectively].

It is difficult to obtain explicit expressions for σ , K_{**} , and C_p in general. We will therefore restrict the frequencies to $\omega\tau_V \leq 1$, when the influence of transient effects of phase interaction in an aerosol on wave dispersion and dissipation can be disregarded in it.

We first consider the simpler special cases of single-component aerosols. For each case we can write explicit relations for the phase velocity and attenuation coefficients as a function of the frequency of the disturbance, the particle size, and the thermophysical properties of the phases.

For a mixture of a gas with solid particles ($k_V = 0$) the expressions for σ , K_{**} , and C_p have the familiar form [12]

$$\sigma = \pi I_G(\omega), \quad K_{**} = (\omega / 2C_1) I_G(\omega), \quad C_p / C_1 = 1 - \text{Re}_G(\omega) / 2,$$

$$I_G(\omega) = m \left\{ \frac{\omega\tau_v}{1 + (\omega\tau_v)^2} + (\gamma_1 - 1) \frac{c_2}{c_1} \frac{\omega\tau_T}{1 + (\omega\tau_T)^2} \right\},$$

$$\text{Re}_G(\omega) = m \left\{ \frac{1}{1 + (\omega\tau_v)^2} + (\gamma_1 - 1) \frac{c_2}{c_1} \frac{1}{1 + (\omega\tau_T)^2} \right\},$$

$$\tau_v = \frac{2}{9} \frac{\rho_2^0 a^2}{\mu_1}, \quad \tau_T = \rho_2^0 c_2 a^2 \left(\frac{1}{3\lambda_1} + \frac{1}{15\lambda_2} \right).$$

Here τ_v is the relaxation time of the phase velocities in quasisteady (Stokes) flow of the gas around the particles, μ is the dynamic viscosity coefficient, and τ_T is a characteristic relaxation time of the temperatures between phases without phase transition on the surface of the droplet.

In the case of a vapor containing droplets ($k_V = 1$), when mass transfer is a quasiequilibrium process ($\tau_\beta = 0$), we can write the following expressions for the coefficients σ , K_{**} , and the velocity C_p :[†]

$$\sigma = \pi \left(\frac{C_p}{C_1} \right)^2 I_V(\omega), \quad K_{**} = \frac{\omega}{2C_1} \left(\frac{C_p}{C_1} \right) I_V(\omega), \quad \left(\frac{C_p}{C_1} \right)^2 = \frac{1}{1 + \text{Re}_V(\omega)}, \quad (3.1)$$

$$I_V(\omega) = I_1 \frac{\omega\tau_{T1}}{1 + (\omega\tau_{T1})^2} + I_2 \frac{\omega\tau_v}{1 + (\omega\tau_v)^2} + I_3 \frac{\omega\tau_{T2}}{1 + (\omega\tau_{T2})^2},$$

$$\text{Re}_V(\omega) = I_1 \frac{1}{1 + (\omega\tau_{T1})^2} + I_2 \frac{1}{1 + (\omega\tau_v)^2} + I_3 \frac{1}{1 + (\omega\tau_{T2})^2},$$

$$I_1 = (\gamma_1 - 1) \left(1 - \frac{\bar{c}_1}{\bar{l}} \right)^2, \quad I_2 = m, \quad I_3 = m \frac{\bar{c}_2}{\bar{l}^2} \quad \left(\bar{c}_j = \frac{c_j}{\gamma_1 R_1}, \quad \bar{l} = \frac{l}{C_1^2} \right),$$

$$\tau_{T1} = \frac{1}{3} \frac{\alpha_1}{\alpha_2} \frac{a^2}{\kappa_1}, \quad \tau_{T2} = \frac{1}{15} \frac{a^2}{\kappa_2}$$

[τ_{Tj} ($j = 1, 2$) is a characteristic heat-transfer time between the j -th phase and the droplet surface, and κ is the thermal diffusivity].

Ordinarily $\bar{c}_2/\bar{l}^2 \ll 1$, so that I_3 and, accordingly, the last term in $\text{Re}_V(\omega)$ is small. At frequencies $\omega\tau_{T2} \ll 1$ ($\omega\tau_v \lesssim 1$) the last term in $I_V(\omega)$ is also small. Consequently, the influence of temperature nonuniformity in the interior of the droplet on wave dispersion and dissipation in single-component aerosols with phase transitions can be disregarded at these frequencies.

In the general case of a two-component aerosol with nonequilibrium mass transfer ($\tau_\beta \neq 0$) the explicit expressions for σ , K_{**} , and C_p have a cumbersome form. However, in the frequency range $\omega\tau_v \lesssim m \ll 1$, where the contribution of phase transition effects to wave dispersion and dissipation is most pronounced [9], these dependences are simplified, and the following expressions can be used for σ , K_{**} , and C_p :

$$\sigma \sim \pi \left(\frac{C_p}{C_1} \right)^2 I_m(\omega), \quad K_{**} \sim \frac{\omega}{2C_1} \left(\frac{C_p}{C_1} \right) I_m(\omega), \quad \left(\frac{C_p}{C_1} \right)^2 \sim \frac{1}{1 + \text{Re}_m(\omega)}, \quad (3.2)$$

$$I_m(\omega) = I_{1m} \frac{\omega\tau_m}{1 + (\omega\tau_m)^2}, \quad \text{Re}_m(\omega) = I_{1m} \frac{1}{1 + (\omega\tau_m)^2},$$

$$I_{1m} = \frac{k_V(\gamma_1 - 1)\gamma_1(\bar{l} - h\bar{c}_1)(\bar{l} - b\bar{c}_1)}{\gamma_1 \bar{l}^2 k_V + (1 - k_V h) b \bar{c}_1},$$

$$b = \frac{R_V}{R_1}, \quad h = (1 - r) + (1 - k_V) \left(\frac{R_V - R_G}{R_1} \right), \quad r = \frac{\rho_1^0}{\rho_2^0}.$$

[†]For a vapor containing droplets the nonequilibrium of the phase interface in the presence of mass transfer asserts itself at rather high frequencies [9]:

$$(\omega\tau_{T2})^{1/2} \gg \frac{\beta\lambda_1}{\lambda_2} \left(\frac{l}{C_1^2} \right) \left(\frac{a}{L} \right), \quad L \sim \kappa_1/C_1.$$

Here τ_m is a characteristic time of nonequilibrium interphase heat and mass transfer in the vapor-gas-droplet mixture at $T_\Sigma \neq T_S$; it is given by the relation

$$\tau_m = \frac{3}{2} \frac{\tau_v}{m} \left[\frac{\gamma_1 \bar{l}^2 k_V Pr_1 + (1 - k_V) \bar{b} \bar{c}_1 Sc_1 + \frac{2\bar{c}_1}{3r} \frac{\tau_\beta}{\tau_v}}{\gamma_1 \bar{l}^2 k_V + (1 - k_V) \bar{b} \bar{c}_1} \right], \quad 0 < m \ll 1, \quad (3.3)$$

in which Pr_1 and Sc_1 are the Prandtl and Schmidt numbers for the host phase. Normally Pr_1 , $Sc_1 \sim 1$, so that for $\tau_\beta = 0$, according to Eq. (3.3), we have $\tau_m \sim \tau_v/m$, i.e., $\tau_v \gg \tau_m$. Note that Eq. (3.2) coincides with Eq. (3.1) at frequencies $\omega\tau_v \ll 1$ in the case of a vapor containing droplets ($k_V = 1$), when $\tau_\beta = 0$.

According to Eqs. (3.2) and (3.3), the frequency dependence of the attenuation per wavelength σ has a maximum at $\omega\tau_m \sim \sqrt{1 + I_{1m}}$. The frequency ω at which σ has an extremum associated with interphase heat and mass transfer depends on the thermophysical parameters of the mixture m , k_V , \bar{l} , and β through τ_m and I_{1m} . For small values of β ($\beta \ll 1$) allowance for the nonequilibrium of mass transfer ($\tau_\beta \neq 0$) can shift the extremum of the function $\sigma \times (\omega)$ toward lower frequencies in connection with the increase in the time τ_m . To estimate the maximum of $\sigma(\omega)$ associated with nonequilibrium interphase heat- and mass-transfer processes, we can use the approximate expression (the error of estimation does not exceed 3-5%)

$$\sigma_{\max} \sim \frac{\pi}{2} \frac{k_V \gamma_1 (\gamma_1 - 1) (\bar{l} - \bar{h} \bar{c}_1) (\bar{l} - \bar{b} \bar{c}_1)}{\gamma_1 \bar{l}^2 k_V + (1 - k_V) \bar{b} \bar{c}_1}. \quad (3.4)$$

Hence it follows that for sufficiently small mass contents m of the disperse phase, such that the function $\sigma(\omega)$ has essentially only one extremum, specifically the extremum associated with interphase heat and mass transfer, the maximum value of the coefficient σ does not depend on m or β . An analysis shows that relation (3.4) is valid with sufficient accuracy over the entire range of vapor concentrations $0 \leq k_V \leq 1$, and it can therefore be used to investigate the function $\sigma_{\max}(k_V)$ over the entire range of the argument. We note that the variation of k_V is associated with the temperature variation of the gaseous phase [$T_1 = T_S(k_V)$], making it difficult to analyze the function $\sigma_{\max}(k_V)$ [or, equivalently, $\sigma_{\max}(T_1)$], since the thermophysical parameters depend on the temperature [$\ell(T_1)$, $c_1(T_1)$, etc.].

The function $\sigma_{\max}(k_V)$ calculated by means of Eq. (3.4) for a mixture of air with water vapor and droplets is shown in Fig. 1 for various initial pressures in the gaseous phase: 1) $p_1 = 0.1$ MPa; 2) 1.0 MPa. The dashed curves illustrate the case $R_V = R_G$ and can be used to assess the influence of differences in the gas constants of the vapor and gas components on the attenuation. Clearly, failure to account for the difference between R_V and R_G makes the attenuation coefficient σ too high. This is attributable to the fact that the parameter $h = 1 + (1 - k_V)(R_V - R_G)/R_1$ reaches its minimum value of unity when $R_V = R_G$. The error incurred by ignoring the difference between R_V and R_G in the determination of σ_{\max} is large in the range of moderate vapor concentrations and increases with the difference between R_V and R_G . It is evident from Fig. 1 that the functions $\sigma_{\max}(k_V)$ have an extremum at practically the same vapor concentration $k_V \sim 0.1$ for the given pressures. Consequently, at pressures $p_1 = 0.1$ -1.0 MPa the wave attenuation in atmospheric fog due to diffusional mass transfer between phases of the mixture is a maximum at $k_V \sim 0.1$.

4. Experimental data on the propagation of low-intensity waves in aerosol fogs in the presence of phase transitions are the most interesting from the standpoint of testing the theory developed here. Just about the only paper on this subject, in which measurements of the attenuation of acoustic disturbances in atmospheric fog are reported, is [3]. In this work a Wilson cloud chamber was used to generate a monodisperse fog with a spatially homogeneous structure. The sizes and concentration of the droplets was determined by optical methods. The experiments were carried out at a mass content of fog droplets $m \sim 10^{-2}$, the wave frequency was 80 Hz, and the droplet diameter was varied from 2 μm to 10 μm .

The objective of the experiments was to investigate the attenuation of harmonic disturbances in the range of dimensionless frequencies $\omega\tau_{T_1} \sim 1$ ($\omega\tau_m \sim 1$, $\omega\tau_v \sim m$), where the influence of heat and mass transfer on the dissipation is a maximum. The dimensionless

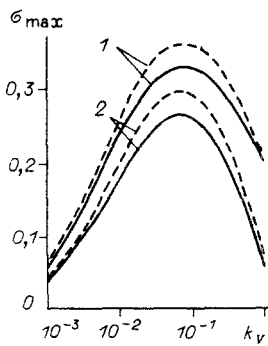


Fig. 1

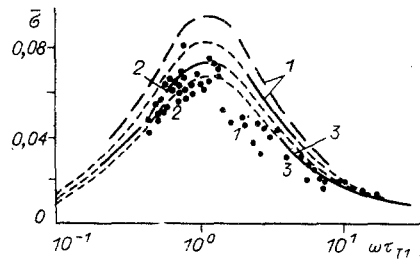


Fig. 2

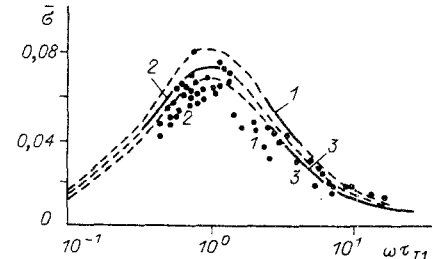


Fig. 3

frequency $\omega\tau_{T1}$ was varied in the range $\sim 0.5-16$ by varying the size of the droplets. The experiments showed that the attenuation of a low-frequency disturbance in an aerosol with phase transitions can be an order of magnitude greater than the attenuation in a gas containing solid particles. The maximum attenuation per wavelength in an aerosol with mass transfer, in contrast with the gas of a gas suspension without phase transitions, is observed at $\omega\tau_{T1} \sim 1$ ($\omega\tau_V \sim m \ll 1$).

In Fig. 2 the theory is compared with the experimental data of Cole and Dobbins [3] on the attenuation per wavelength $\bar{\sigma} = \sigma/\pi$ in a mixture of air with water vapor and droplets. This figure also illustrates the theory developed by the same authors in a previous paper [2]. The three groups of experimental points correspond to three experiments carried out for droplets with different diameters and mass concentrations and at different temperatures (different vapor concentrations in the gaseous phase: 1) $T_0 = 281$ K, $d = 4.36$ μm , $n = 1.71 \cdot 10^5$ cm^{-3} , $k_V = 0.012$; 2) 276 K, 1.84 μm , $1.52 \cdot 10^6$ cm^{-3} , 0.008; 3) 271 K, 4.96 μm , $7.74 \cdot 10^4$ cm^{-3} , 0.006. Here d and n are the diameter and concentration of the droplets per unit volume. The scatter of the experimental data [3] is $\sim 10-15\%$.

The analytical curves of $\bar{\sigma}(\omega\tau_{T1})$ are plotted by means of the dispersion relation [9] on the assumption that mass transfer is a quasiequilibrium process. For a vapor containing droplets with a frozen ($\beta = 0$) or quasiequilibrium ($\beta = \infty$) phase transition the function $\sigma(\omega\tau_V)$ is self-similar and is suitable for particles of any diameter in the range of admissible diameters ($\omega\tau_C \ll 1$, where τ_C is a characteristic time constant of waves whose wavelength is comparable with the distance between inclusions [11]). This is also true in the general case of a vapor-gas-droplet mixture. In these extreme (with respect to β) cases the coefficient σ depends on the droplet radius a only through the dimensionless combinations $\omega\tau_{\mu 1}$, $\omega\tau_{\lambda 1}$, $\omega\tau_{\lambda 2}$, $\omega\tau_d$ [9]: $\tau_{\mu 1}$, $\tau_{\lambda 1}$, $\tau_{\lambda 2}$, $\tau_d \sim a^2$, i.e., $\sigma(\omega, \tau) = \sigma(\omega a^2)$. Moreover, as we showed in Sec. 3 above, in aerosols with small $m \sim 10^{-2}$ and quasiequilibrium phase transition at frequencies $\omega\tau_V \ll 1$ the attenuation σ depends on the mass content of the droplets only through the dimensionless time $\omega\tau_m$ ($\tau_m \sim \tau_{T1} \sim \tau_V/m$), i.e., $\sigma(\omega, \tau, m) \approx \sigma \times (\omega a^2/m)$, m , $\omega\tau_V \ll 1$. We can therefore assume that the function $\sigma(\omega\tau_{T1})$ is approximately universal for suspensions with different droplet diameters and mass concentrations in the range of admissible values. This fact must be taken into account in analyzing the experimental data discussed here.

The slight temperature variation from 271 K to 281 K that takes place in the experiments [3] at a constant initial pressure $p_1 = 0.1$ MPa did not actually affect the diffusion coefficients, the thermal conductivities, the specific heats, or the heats of vaporization. We note, however, that the vapor concentration k_V in the gas mixture varied appreciably for the temperature variation. An analysis shows (see Fig. 1) that when k_V is small, the attenuation coefficient σ is directly proportional to k_V . Consequently, the most important parameter affecting the positions of the curves under the given experimental conditions (Fig. 2) is the vapor concentration k_V in the host phase. Major difficulties were encountered in the precise determination of k_V in the experiments, and so the authors of [3] postulated that the initial state is a thermodynamic equilibrium state: $k_V = k_{VS}(T_0)$.

It is instructive to investigate how the consistency of theory with experiment is influenced by taking into account the difference between the gas constants of the vapor and gas components of the host phase and by taking into account the nonequilibrium of inter-phase mass transfer. Some results of this investigation are shown in Figs. 2 and 3. The dashed curves in Fig. 2 represent the theoretical curves obtained in [2] on the assumption that $R_V = R_G$. The agreement between theory and experiment is improved by allowing for the fact that $R_V \neq R_G$. Theoretical curves corresponding to the three groups of experimental points were calculated on the assumption that $\beta = \infty$ ($\tau_\beta = 0$) and are represented corresponding to the experiments [3] the results of these calculations for $\tau_\beta = 0$ are close to the results obtained by Davidson [7] within the framework of the simpler two-temperature quasi-equilibrium scheme of heat and mass transfer without regard for transient effects.

The investigations showed that allowance for the nonequilibrium of phase transition in aerosol systems with small droplet mass contents m and low vapor concentrations k_V can shift the $\sigma(\omega)$ curve well into the low-frequency range for sufficiently small values of the accommodation coefficient β [9]. The size of the shift increases as β decreases. This situation creates the hypothetical possibility of determining β from the condition of optimum agreement between theory and the experimental data on sound attenuation in aerosols. Unfortunately, the large scatter of the existing experimental data [3] makes it difficult to determine β with any accuracy by this approach.

In Fig. 3 the experimental data [3] are compared with the results of calculations based on the general nonequilibrium theory [9]. Theoretical curves corresponding to $\beta = 0.4$ are plotted. A comparison of Figs. 2 and 3 leads to the conclusion that allowance for the nonequilibrium of phase transition for the given parameters of the mixture has only a slight influence on the maximum value of the attenuation coefficient. However, the agreement of curves 1 and 3 with the corresponding groups of experimental points from [3] is improved somewhat by shifting the nonequilibrium theoretical curves toward lower frequencies. We note that the curves corresponding to $\beta = 1$ differ slightly from the curves for $\beta = 0.4$, but practically coincide with the curves calculated according to the quasiequilibrium scheme ($\beta = \infty$, $\tau_\beta = 0$). Using the accommodation coefficient most often recommended for water, $\beta = 0.04$, somewhat diminishes the agreement of theory with experiment in contrast with $\beta = 0.4$ for curves 2 and 3, but improves the agreement for curve 1. On the whole, the experimental points are situated inside the region bounded by the curves calculated at the limit $\beta = \infty$ and for $\beta = 0.04$.

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MODELING TWO-PHASE FLOWS WITH A PHASE INTERFACIAL SURFACE

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Many important problems for practice of the two-phase flow around bodies whose constituents are air (gas) and water (liquid) can be solved, exactly as in the case of a homogeneous medium, in a boundary-layer approximation that here retains the main structural criteria of a single-phase layer. However, depending on the phase mass relationships (on the degree of water content) important features appear whose crux is the formation and motion of a thin liquid layer over the streamlined surface.

A large number of papers is devoted to the study of stratified flows in order to simulate hydrodynamic processes being realized in different branches of engineering [1-5]. It should be noted that researchers turned the most attention mainly to the examination of two-phase flows with a laminar gas stream [2, 3] while the inhomogeneous structure of the phase separation boundary is not taken into account [4, 5] in the few papers devoted to two-phase flows with a turbulent boundary layer.

It is shown in [6, 7] that the flow of a liquid film subjected to an air stream in a sufficiently broad range of values of the air speed and water mass flow rate in the film is two-parametric in nature and depends on the air Reynolds number $Re_{x,2}$ and on the water film Re_1 . It is found experimentally that the air-water phase interfacial surface is covered in all cases by a complex system of waves whose parameters are random in nature [6]. It is evident here that the air stream parameters influence the film motion while the nature of the liquid flow causes a change in the structure of the air medium.

The mathematical description of the mentioned phenomena is fraught with a number of difficulties including the complexity of taking account of all the processes proceeding in the film and the air stream that results in the necessity to introduce separate assumptions during execution of theoretical computations.

A flow model in the approximation of gas stream incompressibility in the absence of heat and mass transfer turns out to be sufficient for the examination of a number of physical processes. Such problems are encountered, say, in aviation engineering during determination of the aerodynamic characteristics of streamlined structures in the presence of thin liquid films.

A method and model of computing the combined flow of a water film with an air co-stream are proposed in this paper, which are based on the idea of merging the solutions of the air and liquid phase boundary layer equations. The conception of the model is configured in the representation of the laminar nature of the motion in the film and the turbulent nature in the air stream. The condition of continuity of the friction stress and velocity is posed at the boundary separating the two phases and its structure is assumed nonuniform. In the general case the flow is considered gradient and planar.

The air stream characteristics are analyzed by numerical integration of the system of differential equations

$$\bar{u}_2 \frac{\partial \bar{u}_2}{\partial x} + \bar{v}_2 \frac{\partial \bar{u}_2}{\partial y} = -\frac{1}{u_1} \frac{du_1}{dx} + \frac{d\bar{\tau}_2}{dy} - \bar{u}_2 \frac{1}{u_1} \frac{du_1}{dx}; \quad (1)$$

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